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$1/N_c$ Expansion of the Heavy Baryon Isgur-Wise Functions

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Abstract

The $1/N_c$ expansion of the heavy baryon Isgur-Wise functions is discussed. Because of the contracted $SU(2N_f)$ light quark spin-flavor symmetry, the universality relations among the Isgur-Wise functions of $\Lambda_b \to \Lambda_c$ and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ are valid up to the order of $1/N_c^2$.

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Heavy baryons provide us with a testing ground for the Standard Model (SM), especially to Quantum Chromodynamics (QCD) in some aspects. With the accumulation of the experimental data on heavy baryons, some important parameters of SM, for instance the Cabbibo-Kobayashi-Maskawa (CKM) matrix element V_{cb}, can be extracted by comparing experiments with theoretical calculations. The main difficulties in the calculations are due to our poor understanding of the nonperturbative QCD. In this Brief Report, we discuss the $1/N_c$ expansion [1] for the heavy baryon weak decay form factors. We will point out that it can be applied to relate different baryon Isgur-Wise functions with a comparative accuracy.

Heavy baryon weak decays can be systematically studied by the heavy quark effective theory (HQET) [2]. The classification of the form factors parameterizing the hadronic matrix elements of the weak currents is simplified significantly [3]. Under the heavy quark limit, only one universal form factor remains to be determined in the $\Lambda_b \to \Lambda_c$ transition, and two in $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ transitions. These universal form factors are called Isgur-Wise functions. They should be calculated by some nonperturbative method.

Large N_c limit is one of the most important and model-independent method of nonperturbative QCD. Nonperturbative properties of mesons can be observed from the analysis of the planar diagram, and baryons from the Hartree-Fock picture. Recently there are renewed interests in the large N_c application to baryons [4-7]. It is pointed out that there is a contracted SU$(2N_f)$ light quark spin-flavor symmetry in the baryon sector in the large N_c limit. The observation of this light quark spin-flavor symmetry results in many quantitative applications [8-14]. In the large N_c limit, the relations among the baryon Isgur-Wise functions have been studied [11-13]. With the definitions

of

$$
\begin{aligned}
<\Lambda_c(v',s')|\bar{c}\Gamma b|\Lambda_b(v,s)> &= \eta(y)\bar{u}_{\Lambda_c}(v',s')\Gamma u_{\Lambda_b}(v,s)\ , \\
<\Sigma_c^{(*)}(v',s')|\bar{c}\Gamma b|\Sigma_b^{(*)}(v,s)> &= [\xi(y)g_{\mu\nu}+\zeta(y)v_\nu v'_\mu]\bar{u}^\nu_{\Sigma_c^{(*)}}(v',s')\Gamma u^\mu_{\Sigma_b^{(*)}}(v,s)\ ,
\end{aligned}
\tag{1}
$$

where $y = v \cdot v'$, $u^\mu_{\Sigma^*_Q}(v,s)$ is the Rarita-Schwinger spinor and $u^\mu_{\Sigma_Q}(v,s)$ is defined by

$$
u^\mu_{\Sigma_Q}(v,s) = \frac{(\gamma^\mu + v^\mu)\gamma_5}{\sqrt{3}} u_{\Sigma_Q}(v,s)\ , \tag{2}
$$

the Isgur-Wise functions $\eta(y)$, $\xi(y)$ and $\zeta(y)$ have the following large N_c relations [11, 12],

$$
\eta(y) = \xi(y) = -(y+1)\zeta(y)\ . \tag{3}
$$

We study the $1/N_c$ expansion of the relations among the baryon Isgur-Wise functions. It is interesting because of the observation of the light quark spin-flavor symmetry in the baryon sector in the large N_c limit. It is heuristic to illustrate the large N_c baryon Isgur-Wise functions in a naive way which can be directly applied to the discussions of the $1/N_c$ corrections. Under the heavy quark limit, the heavy quark in the heavy hadron has fixed velocity which is identical with the heavy hadron velocity. And the heavy quark spin decouples from its strong interaction with the light quark system in the hadron. The light quark system cannot see any properties of the heavy quark except its velocity. In this case, the spin J_l and isospin I_l of the light quark system become good quantum numbers to describe the heavy baryons in which, quarks have no orbital angular momentum excitations in the constituent picture, and there are only two flavors of light quarks. For the number of colors being N_c, the baryons we are interested in are $(I_l, J_l) = (0,0), (1,1), ..., (\frac{N_c-1}{2}, \frac{N_c-1}{2})$. When the velocity of the heavy quark changes from v to v' due to weak decay, the brown muck has to undergo a transition through the strong interaction from the heavy quark. The Isgur-Wise functions defined in Eqs. (1) and (2) just measure the amplitudes of the brown muck transfers. They cannot be determined from the HQET further, however. It is at this stage, the large N_c method is applied. In the large N_c limit, there is the SU(4) light

quark spin-flavor symmetry for baryons [4]. During the transition, the spin of any light quark in the brown muck is conserved. In other words, the light quark spin individually decouples from the strong interaction in the brown muck transition. The Isgur-Wise functions are independent of the light quark spin configuration in the brown muck in the large N_c limit, and therefore deserve an SU(4) expansion. At the leading order of the SU(4) expansion, the relations given by Eq. (3) still hold. This is the essential point in deriving the large N_c universality of the baryon Isgur-Wise functions for the $\Lambda_b \to \Lambda_c$ and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ decays in Ref. [12] based on the SU(4) symmetry.

Let us go further to the corrections of this contracted SU(4) light quark spin-flavor symmetry result. They can be considered straightforwardly along above discussion. The corrections only appear in the finite N_c case. We note the baryon spectrum has the relation $I_l = J_l$. Therefore the Isgur-Wise functions have the following expansions,

$$\begin{aligned} \eta(y) &= \tilde{\eta}_0(y) , \\ \xi(y) &= \tilde{\eta}_0(y) + \tilde{\xi}(y)\frac{J_l^2}{N_c^2} , \\ -(y+1)\zeta(y) &= \tilde{\eta}_0(y) + \tilde{\zeta}(y)\frac{J_l^2}{N_c^2} , \end{aligned} \tag{4}$$

where $\tilde{\eta}_0(y)$ is the leading SU(4) symmetry result which is independent of the brown muck spin or isospin. $\tilde{\xi}(y)$ and $\tilde{\zeta}(y)$ parameterize the SU(4) breaking effects, and are normalized to be order 1 in the large N_c limit. The factor N_c^2 should be there so as to keep the N_c scaling for the Isgur-Wise functions. In the extreme case while in the baryon all the light quark spins align in the same direction, J_l^2 scales as $N_c^2/4$. Only by dividing a factor N_c^2, have the terms proportional to J_l^2 in above equation the right N_c scaling. Note there is no term which has linear dependence on J_l in the corrections. This is simply because there is no way to combine J_l with $(v - v')^\mu$ into a Lorentz and CP invariant quantity. Generally, the Isgur-Wise function should depend on J_l/N_c or I_l/N_c of the brown muck undergoing the transition. However it is interesting to note that the Isgur-Wise function $\eta(y)$ does not have any corrections in the SU(4) expansion because Λ_Q baryon is a SU(4) singlet. From Eq. (4), we see that the SU(4) symmetry

relations (3) are valid up to the order of $1/N_c^2$.

It is helpful for the understanding to compare this framework with the heavy quark Skyrme model [15, 16] which is often believed to be the large N_c HQET. The model predicted an exponential form for the Isgur-Wise function $\eta(y)$ [17]. But the way to calculate its $1/N_c$ corrections is not available. On the other hand, as is well-known, there are $O(1/N_c)$ contributions in $\tilde{\eta}_0(y)$, we have no knowledge about $\tilde{\eta}_0(y)$ itself, however the predicted universality relations of the Isgur-Wise function, Eq. (4), are valid up to the order of $1/N_c^2$. (The analogous situation holds for the case of baryon masses [9, 14].) If we focus on the relation among the Isgur-Wise functions, the SU(4) expansion provides a better framework. In the near future, the Isgur-Wise function $\eta(y)$ can be extracted from the experimental data of the $\Lambda_b \to \Lambda_c$ semileptonic decay. With this information, the weak decays $\Omega_b^{(*)} \to \Omega_c^{(*)}$ can be predicted to a comparatively accurate level in the chiral SU(3) symmetry limit.

Finally let us add a remark on the $1/m_Q$ expansion for the form factors. HQET is a systematic method for the expansion [18]. What we have discussed in this note is just the leading order of heavy quark expansion. To the order of $1/m_Q$, there is an additional universal form factor for the $\Lambda_b \to \Lambda_c$ semileptonic decay. This form factor vanishes in the limit of $N_c \to \infty$. Therefore when we extract $\eta(y)$ from the decay of $\Lambda_b \to \Lambda_c$, it subjects to an uncertainty of $\Lambda_{QCD}/N_c m_Q$. This uncertainty can be the same order of $1/N_c^2$ numerically, therefore does not spoil the accuracy we hoped to achieve.

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